THEORY OF ULTRASONIC WAVE PROPAGATION IN POLYCRYSTALLINE MEDIA

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A method described in [1] is used to investigate the propagation of long and short ultrasonic waves in a polycrystalline medium having orthorhombic symmetry. The attenuation and dispersion of the velocity of elastic waves due to wave scattering by inhomogeneities are calculated. The special cases of tetragonal, hexagonal, and cubic symmetry are analyzed. The results are compared with the data of [2] on scattering in the long-wave approximation as well as with the results of [1, 3] on scattering in higher-symmetry polycrystalline media.

1. The propagation of elastic waves in inhomogeneous media is attended by scattering at structural inhomogeneities and a corresponding dispersion of the propagation velocity. This effect was first analyzed within the framework of the theory of stochastic functions by Lifshits and Parkhomovskii [1] for polycrystalline materials having a cubic structure. Later the scattering of waves in polycrystalline media was investigated for cases of lower symmetry: hexagonal [3] and orthorhombic [2]. In the latter case, however, only the wave attenuation in the long-wave approximation was calculated, where the wavelength greatly exceeds the characteristic dimensions of the crystal grains. The development of gigahertz techniques, on the other hand [3], requires the analysis of the short-wave asymptotic behavior as well. In the present article, therefore, we calculate the scattering coefficient and velocity dispersion for ultrasound propagating in orthorhombic polycrystalline media for both short and long waves, relying on the method of [1].

The analysis is based on the calculation of the second-rank tensor C_{il}:

$$C_{il} = C_{iklm} l_k l_m \tag{1.1}$$

$$C_{iklm} = A^{ikpq}_{stlm} I_{pqst} \tag{1.2}$$

Here A^{ikpq}_{stlm} denotes the tensor part of the binary correlation tensor elastic modulus λ_{ikpq}:

$$A^{ikpq}_{stlm}(\mathbf{r}) = \langle [\lambda_{ikpq}(\mathbf{r}) - \langle\lambda_{ikpq}\rangle] [\lambda_{stlm}(\mathbf{r}) - \langle\lambda_{stlm}\rangle] \rangle, \tag{1.3}$$

$l_i = q_i/q$ is the unit vector in the direction of wave propagation, C_{iklm} is the correlation correction to the average tensor elastic modulus, and

$$I_{pqst} = K_{pqst} + iL_{pqst} = \int G_{ps}(\mathbf{r}) [\varphi(\mathbf{r}) \cos \mathbf{qr}]_{,qt} d\mathbf{r} \tag{1.4}$$

The subscripts following the comma in this case signify differentiation with respect to the indicated coordinates, G_{ps} is the Green tensor of the wave equation for a medium having the average elastic moduli $\langle\lambda_{iklm}\rangle$, and $\varphi(\mathbf{r})$ is the coordinate part of the correlation tensor (1.3).

2. Hereinafter we indicate variables referring to long ($qa < 1$) and short ($qa \gg 1$) waves by subscript minus and plus signs, respectively. Here a is the correlation scale.

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Expressions for I_{pqst} in the long- and short-wave approximations are given in [1]. Correcting some misprints, we now write the appropriate equations:

$$K^{-}_{pqst} = K^{\circ}_{pqst} + \langle a^2\rangle \omega^2 K^{I}_{pqst} \tag{2.1}$$

$$K^{\bullet}_{pqst} = \frac{4\pi}{15}\{g_0[2(\delta_{pt}\delta_{sq} + \delta_{pq}\delta_{st}) - 3\delta_{ps}\delta_{tq}] - 5h_0\delta_{ps}\delta_{tq}\} \tag{2.2}$$

$$\begin{aligned} K^{I}_{pqst} = \frac{1}{c^2}\Big\{ & \frac{g_0}{105}[3\delta_{ps}\delta_{tq} - 4(\delta_{pt}\delta_{sq} + \delta_{pq}\delta_{st}) + 3(2l_q l_s\delta_{pt} + 2l_p l_t\delta_{sq} + \\ & + 2l_q l_p\delta_{st} + 2l_s l_t\delta_{pq} + 2l_p l_s\delta_{tq} - 5l_q l_t\delta_{ps})] + \frac{h_0}{15}(\delta_{ps}\delta_{tq} - 3l_q l_t\delta_{ps})\Big\} + \\ & + 2\Big\{\frac{g_2}{15}[4(\delta_{pt}\delta_{sq} + \delta_{st}\delta_{pq}) - \delta_{ps}\delta_{qt}] + \frac{h_2}{3}\delta_{qt}\delta_{ps}\Big\} \end{aligned} \tag{2.3}$$

$$L^{-}_{pqst} = \langle a^3\rangle \omega^3[(\delta_{pq}\delta_{st} + \delta_{pt}\delta_{sq})g_3 + \delta_{ps}\delta_{qt}2h_3] \tag{2.4}$$

$$(I^{t}_{pqst})^{+} = -\frac{l_q l_t}{\rho}\Big\{-\frac{(l_p l_s - \delta_{ps})}{4c_t^2} + \frac{l_p l_s}{c_l^2 - c_t^2} + i\frac{(l_p l_s - \delta_{ps})\langle a\rangle q_t}{2c_t^2}\Big\} \tag{2.5}$$

$$(I^{l}_{pqst})^{+} = -\frac{l_q l_t}{\rho}\Big\{-\frac{l_p l_s - \delta_{ps}}{c_t^2 - c_l^2} + \frac{l_p l_s}{4c_l^2} - i\frac{l_p l_s}{2c_l^2}\langle a\rangle q_l\Big\} \tag{2.6}$$

in which

$$\langle a^2\rangle = 4\pi\int_0^\infty \varphi(r)\,r\,dr, \quad \langle a^3\rangle = 4\pi\int_0^\infty \varphi(r)\,r^2dr, \quad \langle a\rangle = \int_0^\infty \varphi(r)\,dr \tag{2.7}$$

Here $\omega = 2\pi f$ is the cyclic frequency, and c_l and c_t are the longitudinal and transverse wave velocities of sound in the Voigt approximation:

$$c_l^2 = \frac{1}{\rho}\langle\alpha + 2\beta\rangle, \quad c_t^2 = \frac{1}{\rho}\langle\beta\rangle \tag{2.8}$$

The quantities g_i and h_i are defined by Eqs. (36) of [1].

Substituting expressions (2.1)-(2.6) into Eq. (1.2), we find

$$\begin{aligned} C^{-}_{il} = {} & \frac{4\pi}{15}[g_0(2l_k l_m A^{sslm}_{ikpp} - l_k l_m A^{pqlm}_{ikpq}) - 5h_0 l_k l_m A^{pqlm}_{ikpq}] + \langle a^2\rangle\omega^2\Big\{\frac{g_0}{105c^2}[-l_k l_m A^{pqlm}_{ikpq} - 4l_k l_m A^{sslm}_{ppik} + 3l_k l_m l_q l_s A^{splm}_{ikpq} + \\ & + 12 l_k l_m l_p l_q A^{sslm}_{ikpq}] + \frac{h_0}{15c^2}[l_k l_m A^{pqlm}_{ikpq} - 3l_k l_m l_q l_s A^{splm}_{ikpq}] + \frac{2g_2}{15}[3l_k l_m A^{pqlm}_{ikpq} + 4l_k l_m A^{sslm}_{ikpp}] + \frac{2h_2}{3}l_k l_m A^{pqlm}_{ikpq}\Big\} + \\ & + i\langle a^3\rangle\omega^3[l_k l_m A^{sslm}_{ikpq}g_3 + l_k l_m A^{pqlm}_{ikpq}(g_3 + 2h_3)] \end{aligned} \tag{2.9}$$

$$(C^{t}_{il})^{+} = -l_k l_m l_s l_t l_q l_p A^{stlm}_{ikpq}\left(\frac{5c_t^2 - c_l^2}{4\rho c_t^2(c_l^2 - c_t^2)} + i\frac{\langle a\rangle\omega}{2\rho c_t^3}\right) - l_k l_m l_t l_q A^{ptlm}_{ikpq}\left(\frac{1}{4\rho c_t^2} - i\frac{\langle a\rangle\omega}{2\rho c_t^3}\right) \tag{2.10}$$

$$(C_{il}^{l})^{+} = -l_k l_m l_s l_t l_q l_p A^{stlm}_{ikpq}\left(\frac{5c_l^2 - c_t^2}{4\rho c_l^2(c_l^2 - c_t^2)} - i\frac{\langle a\rangle\omega}{2\rho c_l^3}\right) + l_k l_m l_t l_q A^{ptlm}_{ikpq}\frac{1}{\rho(c_l^2 - c_t^2)} \tag{2.11}$$

The quantities C^{t}_{il} and C^{l}_{il} are found from (2.9) by the appropriate substitution $c \to c_t$ or $c \to c_l$.

3. In order to calculate the various contractions of the autocorrelation tensor A^{stlm}_{ikpq} that determine the tensor C_{il} according to Eqs. (2.9)-(2.11), we use the explicit value found in [5] for this tensor in the case of orthorhombic symmetry (allowing for the fact that the coefficient of $p_{\lambda\lambda}\delta_{ijkl}\delta_{pqrs}$ is equal to $-21/5$). After some simple but laborious computations we obtain

$$l_{kmpq}A_{ikpq}^{sslm} = A_1 l_{il} + A_2\delta_{il}, \quad l_{km}A_{ikpp}^{sslm} = A_3 l_{il} + A_4\delta_{il}$$

$$l_{kmqs}A_{ikpq}^{splm} = A_5 l_{il} + A_6\delta_{il}, \quad l_{km}A_{ikpq}^{pqlm} = A_7 l_{il} + A_8\delta_{il} \tag{3.1}$$

$$l_{kmstpq}A_{ikpq}^{stlm} = A_9 l_{il} + A_{10}\delta_{il}$$

where

$$l_{kmp\ldots} = l_k l_m l_p \ldots$$

$$A_1 = 5/3A_2 = 1/63\,(3P_{\lambda\lambda} + 16P_{\lambda\mu} + 26P_{\lambda\nu} + 21P_{\mu\mu} + 70P_{\mu\nu} + 56P_{\nu\nu})$$

$$A_3 = 1/3A_4 = 1/45\,(P_{\lambda\lambda} + 6P_{\lambda\mu} + 8P_{\lambda\nu} + 9P_{\mu\mu} + 24P_{\mu\nu} + 16P_{\nu\nu})$$

$$A_5 = \frac{1}{15\cdot 7!!}\left(112P_{\lambda\lambda} + 12\sum_n \lambda^{(n)}\lambda^{(n)} + 570P_{\lambda\mu} + 1070P_{\lambda\nu} + 665P_{\mu\mu} + 2660P_{\mu\nu} + 2415P_{\nu\nu}\right)$$

$$A_6 = \frac{1}{15\cdot 7!!}\left(24\sum_n \lambda^{(n)}\lambda^{(n)} + 14P_{\lambda\lambda} + 90P_{\lambda\mu} + 250P_{\lambda\nu} + 105P_{\mu\mu} + 420P_{\mu\nu} + 665P_{\nu\nu}\right) \tag{3.2}$$

$$A_7 = \frac{1}{15\cdot 5!!}\left(3\sum_n \lambda^{(n)}\lambda^{(n)} + 18P_{\lambda\lambda} + 110P_{\lambda\mu} + 160P_{\lambda\nu} + 165P_{\mu\mu} + 440P_{\mu\nu} + 340P_{\nu\nu}\right)$$

$$A_8 = \frac{1}{15\cdot 5!!}\left(9\sum_n \lambda^{(n)}\lambda^{(n)} + 4P_{\lambda\lambda} + 30P_{\lambda\mu} + 80P_{\lambda\nu} + 45P_{\mu\mu} + 120P_{\mu\nu} + 220P_{\nu\nu}\right)$$

$$A_9 = \frac{1}{15\cdot 7!!}\left(6\sum_n \lambda^{(n)}\lambda^{(n)} + 81P_{\lambda\lambda} + 390P_{\lambda\mu} + 780P_{\lambda\nu} + 455P_{\mu\mu} + 1820P_{\mu\nu} + 1820P_{\nu\nu}\right)$$

$$A_{10} = \frac{1}{15\cdot 7!!}\left(10\sum_n \lambda^{(n)}\lambda^{(n)} + 15P_{\lambda\lambda} + 90P_{\lambda\mu} + 180P_{\lambda\nu} + 105P_{\mu\mu} + 420P_{\mu\nu} + 420P_{\nu\nu}\right)$$

$$P_{\lambda\mu} \equiv \frac{1}{2}\left(3\sum_n^3 \lambda^{(n)}\mu^{(n)} - \sum_n^3 \lambda^{(n)}\sum_m^3 \mu^{(m)}\right) \tag{3.3}$$

The elastic coefficients $\lambda^{(n)}$, $\mu^{(n)}$, and $\nu^{(n)}$ are expressed in terms of the matrix elastic constants by the following equations given in [5]:

$$\begin{aligned}
\lambda^{(1)} &= c_{11} + c_{23} + 2c_{44} - (c_{12} + c_{13} + 2c_{55} + 2c_{66})\\
\lambda^{(2)} &= c_{22} + c_{13} + 2c_{55} - (c_{12} + c_{23} + 2c_{44} + 2c_{66})\\
\lambda^{(3)} &= c_{33} + c_{12} + 2c_{66} - (c_{13} + c_{23} + 2c_{44} + 2c_{55})\\
2\mu^{(1)} &= c_{12} + c_{13} - c_{23}, \quad 2\mu^{(2)} = c_{12} + c_{23} - c_{13}\\
2\mu^{(3)} &= c_{13} + c_{23} - c_{12}, \quad 2\nu^{(1)} = c_{55} + c_{66} - c_{44}\\
2\nu^{(2)} &= c_{44} + c_{66} - c_{55}, \quad 2\nu^{(3)} = c_{44} + c_{55} - c_{66}
\end{aligned} \tag{3.4}$$

Now, if we substitute expressions (3.1) into Eqs. (2.9)-(2.11), we can reduce each of the latter equations to the form

$$C_{il} = [\alpha^*(\omega) + \beta^*(\omega)]\, l_i l_l + \beta^*(\omega)\,\delta_{il} \tag{3.5}$$

The real parts α_1 and β_1 and imaginary parts α_2 and β_2 of the effective Lamé coefficients $\alpha^*(\omega)$ and $\beta^*(\omega)$ determine the absorption coefficients and velocity dispersion:

$$\gamma_t(\omega) = \frac{\omega\beta_2(\omega)}{2\rho c_t^3}, \quad \gamma_l(\omega) = \frac{\omega\,[\alpha_2(\omega) + 2\beta_2(\omega)]}{2\rho c_l^3} \tag{3.6}$$

$$v_t(\omega) = c_t\left[1 + \frac{\beta_1(\omega) + \omega\,(d\beta_1(\omega)/d\omega)}{2\rho c_t^2}\right]$$

$$v_l(\omega) = c_l\left[1 + \frac{\alpha_1(\omega) + 2\beta_1(\omega) + \omega\,(d/d\omega)\,(\alpha_1(\omega) + 2\beta_1(\omega))}{2\rho c_l^2}\right] \tag{3.7}$$

Hence, inserting the explicit values of α_i and β_i we find

$$\gamma_l^- = \frac{4\pi^3 \langle a^3\rangle f^4}{5\rho^2 c_l^3}\left(\frac{B_1}{2c_l^5} + \frac{B_2}{c_t^5}\right), \quad \gamma_t^- = \frac{4\pi^3 \langle a^3\rangle f^4}{5\rho^2 c_t^3}\left(\frac{B_3}{2c_l^5} + \frac{B_4}{c_t^5}\right) \tag{3.8}$$

$$v_l^-(f) = c_l(1 - a_1 - a_2\langle a^2\rangle f^2), \quad v_t^- = c_t(1 - a_3 - a_4\langle a^2\rangle f^2) \tag{3.9}$$

$$\gamma_l^+ = \frac{\pi^2\langle a\rangle f^2}{\rho^2 c_l^6} B_9, \quad \gamma_t^+ = \frac{\pi^2\langle a\rangle f^2}{\rho^2 c_t^6} B_{10} \tag{3.10}$$

$$v_l^+ = c_l(1 + a_5), \quad v_t^+ = c_t(1 + a_6) \tag{3.11}$$

$$\begin{gathered} a_1 = \frac{11}{10\rho^2 c_l^2}\left(\frac{B_1}{c_l^2} + \frac{2B_2}{c_t^2}\right), \quad a_2 = \frac{\pi}{70\rho^2 c_l^2}\left(\frac{B_5}{c_l^4} + \frac{B_6}{c_l^2 c_t^2} + \frac{42B_2}{c_t^4}\right) \\ a_3 = \frac{1}{10\rho^2 c_t^2}\left(\frac{B_3}{c_l^2} + \frac{2B_4}{c_t^2}\right), \quad a_4 = \frac{\pi}{70\rho^2 c_t^2}\left(\frac{21B_3}{c_l^4} + \frac{2B_7}{c_l^2 c_t^2} + \frac{B_8}{c_t^4}\right) \\ a_5 = \frac{4c_l^2 B_{11} - (5c_l^2 - c_t^2) B_{12}}{8\rho^2 c_l^4 (c_l^2 - c_t^2)}, \quad a_6 = -\frac{(5c_t^2 - c_l^2) B_{13} + (c_l^2 - c_t^2) B_{14}}{8\rho^2 c_t^4 (c_l^2 - c_t^2)} \end{gathered} \tag{3.12}$$

Here

$$\begin{gathered} B_1 = {}^1/_3(2A_8 + 2A_7 + {}^4/_3A_4), \quad B_2 = {}^1/_6(3A_8 + 3A_7 - {}^4/_3A_4) \\ B_3 = {}^1/_3(2A_8 + A_4), \quad B_4 = {}^1/_6(3A_8 - A_4) \\ B_5 = 10(A_7 + A_8) + 12(A_5 + A_6) + {}^{20}/_3A_4 + 16A_2; \quad B_6 = 9(A_5 + A_6) - 3(A_7 + A_8) + {}^8/_3A_4 - 16A_2 \\ B_7 = 6A_6 + 3A_2 - 2A_8 - A_4, \quad B_8 = 18A_8 + 9A_6 - 5A_4 - 6A_2 \\ B_9 = A_9 + A_{10}, \quad B_{10} = A_6 - A_{10} \\ B_{11} = A_5 + A_6, \quad B_{12} = B_9 \\ B_{13} = A_{10}, \quad B_{14} = A_6 \end{gathered} \tag{3.13}$$

The constants A_{ik} are calculated according to Eqs. (3.2)-(3.4). The average Lamé constants, which according to (2.8) determine the ultrasonic velocities c_l and c_t, are equal to

$$\langle\alpha\rangle = \sum_n^3 ({}^1/_{15}\lambda^{(n)} + {}^2/_3\mu^{(n)}), \quad \langle\beta\rangle = \sum_n^3 ({}^1/_{15}\lambda^{(n)} + {}^2/_3\nu^{(n)}) \tag{3.14}$$

The elastic coefficients $\lambda^{(n)}$, $\mu^{(n)}$, and $\nu^{(n)}$ are given by relations (3.4).

The foregoing result (3.8)-(3.12) can be written, in the notation of [2], in terms of the double-index elastic constants c_{ik} by means of relations (3.13), (3.2), (3.3), and (3.4) as follows:

$$\begin{gathered} B_1 = {}^8/_{675}\, P^2 + {}^8/_{135}\,(4b_1 + 2b_2 + 3b_3 + b_4) \\ B_2 = B_3 = {}^2/_{225}\, P^2 + {}^1/_{135}\,(24b_1 + 7b_2 + 13b_3 + b_4) \\ B_4 = {}^1/_{150}\, P^2 + {}^1/_{90}\,(12b_1 + b_2 + 4b_3 - 2b_4) \\ B_5 = {}^8/_{1575}\,(53P^2 + 1000b_1 + 665b_2 + 630b_3 + 280b_4 + 120b_5 - 180b_6) \\ B_6 = 4B_7 = {}^4/_{1575}\,(6P^2 + 30b_1 + 70b_2 - 140b_3 - 35b_4 + 75b_5 - 165b_6) \\ B_8 = {}^1/_{105}\,(30P^2 + 591b_1 + 49b_2 + 196b_3 - 98b_4 - 3b_5 - 6b_6) \\ B_9 = B_{12} = {}^{16}/_{4725}\,(P^2 + 20b_1 + 20b_2 + 5b_3 + 5b_4 + 10b_5 - 10b_6) \\ B_{10} = {}^1/_{4725}\,(14P^2 + 295b_1 + 25b_2 + 100b_3 - 50b_4 + 5b_5 + 10b_6) \\ B_{11} = {}^2/_{1575}\,(6P^2 + 100b_1 + 75b_2 + 25b_3 + 15b_4 + 30b_5 - 50b_6) \\ B_{13} = {}^1/_{945}\,(2P^2 + 28b_1 + 13b_2 + 7b_3 + b_4 + 2b_5 - 14b_6) \\ B_{14} = {}^1/_{1575}\,(8P^2 + 145b_1 + 30b_2 + 45b_3 - 15b_4 + 5b_5 - 20b_6) \end{gathered} \tag{3.15}$$

where

$$\begin{aligned} P &= (c_{11} + c_{22} + c_{33}) - (c_{12} + c_{13} + c_{23}) - 2(c_{44} + c_{55} + c_{66}) \\ b_1 &= (c_{44} + c_{55} + c_{66})^2 - 3\,(c_{44}c_{55} + c_{55}c_{66} + c_{66}c_{44}) \\ b_2 &= (c_{11} + c_{22} + c_{33})^2 - 3\,(c_{11}c_{22} + c_{22}c_{33} + c_{33}c_{11}) \\ b_3 &= (c_{12} + c_{13} + c_{23})^2 - 3(c_{12}c_{13} + c_{13}c_{23} + c_{23}c_{12}) \end{aligned} \tag{3.16}$$

$$b_4 = c_{11}(c_{12} + c_{13} - 2c_{23}) + c_{22}(c_{12} + c_{23} - 2c_{13}) + c_{33}(c_{13} + c_{23} - 2c_{12})$$
$$b_5 = c_{11}(c_{55} + c_{66} - 2c_{44}) + c_{22}(c_{44} + c_{66} - 2c_{55}) + c_{33}(c_{44} + c_{55} - 2c_{66})$$
$$b_6 = c_{23}(c_{55} + c_{66} - 2c_{44}) + c_{13}(c_{44} + c_{66} - 2c_{55}) + c_{12}(c_{44} + c_{55} - 2c_{66})$$

and

$$\langle\alpha\rangle = {}^1/_{15}\ [(c_{11} + c_{22} + c_{33}) + 4(c_{12} + c_{13} + c_{23}) - 2(c_{44} + c_{55} + c_{66})]$$
$$\langle\beta\rangle = {}^1/_{15}\,[(c_{11} + c_{22} + c_{33}) - (c_{12} + c_{13} + c_{23}) + 3(c_{44} + c_{55} + c_{66})] \qquad (3.17)$$

4. The expressions found above for the long-wave scattering coefficients (3.8) exactly coincide with the results of [2] (see also [6]) if we assume that $\langle a^3\rangle = T$, where T is the average volume of the actual grain. This fact is evident from (3.8) if we evaluate $B_1, \ldots, B_4$ according to (3.15). In order to express the dispersion and scattering coefficients for short waves [see expressions (3.9)-(3.12), (3.15), and (3.16)] we have augmented the parameters P, $b_1, \ldots, b_4$ of [2] with two additional parameters b_5 and b_6.

For tetragonal symmetry ($c_{22} = c_{11}$, $c_{23} = c_{13}$, $c_{55} = c_{44}$) the results can be deduced from Eqs. (3.8)-(3.12), (3.15), and (3.16) by the use of the relations

$$P = \lambda_3 + 3\lambda_6,\quad b_1 = \lambda_5^2,\ b_2 = (\lambda_3 + 2\lambda_4 + 4\lambda_5)^2,\quad b_3 = \lambda_4^2 \qquad (4.1)$$
$$b_4 = 2\lambda_4(\lambda_3 + 2\lambda_4 + 4\lambda_5),\quad b_5 = 2\lambda_5(\lambda_3 + 2\lambda_4 + 4\lambda_5),\ b_6 = -2\lambda_4\lambda_5$$

in which λ_i are the one-index elastic constants described, according to [5], by the expressions

$$\lambda_1 = c_{12},\ \lambda_2 = c_{66},\ \lambda_3 = c_{33} - c_{11} - 2(c_{13} - c_{12} + 2c_{44} - 2c_{66}),$$
$$\lambda_4 = c_{13} - c_{12},\ \lambda_5 = c_{44} - c_{66},\ \lambda_6 = c_{11} - c_{12} - 2c_{66} \qquad (4.2)$$

Now, for the coefficients B_{ik} we have

$$B_1 = {}^8/_{675}(11\lambda_3^2 + 75\lambda_4^2 + 180\lambda_5^2 + 9\lambda_6^2 + 50\lambda_3\lambda_4 + 80\lambda_3\lambda_5 + 6\lambda_3\lambda_6 + 200\lambda_4\lambda_5)$$

$$B_2 = B_3 = {}^1/_{675}\ (41\lambda_3^2 + 225\lambda_4^2 + 680\lambda_5^2 + 54\lambda_6^2 + 150\lambda_3\lambda_4 + 280\lambda_3\lambda_5 + 36\lambda_3\lambda_6 + 600\lambda_4\lambda_5)$$

$$B_4 = {}^1/_{450}(8\lambda_3^2 + 140\lambda_5^2 + 27\lambda_6^2 + 40\lambda_3\lambda_5 + 18\lambda_3\lambda_6)$$

$$B_5 = {}^8/_{1575}(718\lambda_3^2 + 4410\lambda_4^2 + 12600\lambda_5^2 + 477\lambda_6^2 + 3220\lambda_3\lambda_4 + 5560\lambda_3\lambda_5 + 318\lambda_3\lambda_6 + 13720\lambda_4\lambda_5)$$

$$B_6 = 4B_7 = {}^8/_{1575}(38\lambda_3^2 + 875\lambda_5^2 + 27\lambda_6^2 + 105\lambda_3\lambda_4 + 355\lambda_3\lambda_5 + 18\lambda_3\lambda_6 + 735\lambda_4\lambda_5)$$

$$B_8 = {}^1/_{105}\ (79\lambda_3^2 + 1351\lambda_5^2 + 270\lambda_6^2 + 386\lambda_3\lambda_5 + 180\lambda_3\lambda_6)\,. \qquad (4.3)$$

$$B_9 = B_{12} = {}^{16}/_{4725}\ (21\lambda_3^2 + 105\lambda_4^2 + 420\lambda_5^2 + 9\lambda_6^2 + 90\lambda_3\lambda_4 + 180\lambda_3\lambda_5 + 6\lambda_3\lambda_6 + 420\lambda_4\lambda_5)$$

$$B_{10} = {}^1/_{1575}\ (13\lambda_3^2 + 245\lambda_5^2 + 42\lambda_6^2 + 70\lambda_3\lambda_5 + 28\lambda_3\lambda_6)$$

$$B_{11} = {}^2/_{1575}\ (81\lambda_3^2 + 385\lambda_4^2 + 1540\lambda_5^2 + 54\lambda_6^2 + 330\lambda_3\lambda_4 + 660\lambda_3\lambda_5 + 36\lambda_3\lambda_6 + 1540\lambda_4\lambda_5)$$

$$B_{13} = {}^1/_{315}(5\lambda_3^2 + 21\lambda_4^2 + 84\lambda_5^2 + 6\lambda_6^2 + 18\lambda_3\lambda_4 + 36\lambda_3\lambda_5 + 4\lambda_3\lambda_6 + 84\lambda_4\lambda_5)$$

$$B_{14} = {}^1/_{1575}(38\lambda_3^2 + 105\lambda_4^2 + 665\lambda_5^2 + 72\lambda_6^2 + 90\lambda_3\lambda_4 + 250\lambda_3\lambda_5 + 48\lambda_3\lambda_6 + 420\lambda_4\lambda_5)$$

In this case

$$\langle\alpha\rangle = {}^1/_{15}\ (15\lambda_1 + \lambda_3 + 10\lambda_4 + 3\lambda_6),\ \langle\beta\rangle = {}^1/_{15}\ (15\lambda_2 + \lambda_3 + 10\lambda_5 + 3\lambda_6) \qquad (4.4)$$

The results for hexagonal symmetry can be obtained from Eqs. (3.8)-(3.12), (4.3), and (4.4) by setting $\lambda_6 = 0$ in Eqs. (4.3), (4.4), and (4.2). For the long-wave approximation of longitudinal and transverse modes as well as for the short-wave approximation of longitudinal modes we arrive at the familiar expressions for the absorption coefficient [3] (see also [6]).

The results for cubic symmetry ($c_{13} = c_{12}$, $c_{66} = c_{44}$, $c_{33} = c_{11}$) are obtained by recognizing that in this case $\lambda_3 = \lambda_4 = \lambda_5 = 0$. The calculations yield

$$\gamma_l^- = \frac{8\pi^3\lambda_6^2 \langle a^3\rangle f^4}{375\rho^2 c_l^3}\left(\frac{2}{c_l^5} + \frac{3}{c_t^5}\right), \quad \gamma_t^- = \frac{2\pi^3\lambda_6^2 \langle a^3\rangle f^4}{125\rho^2 c_t^3}\left(\frac{2}{c_l^5} + \frac{3}{c_t^5}\right) \tag{4.5}$$

$$v_l^-(f) = c_l(1 - a_1 - a_2\langle a^2\rangle f^2), \quad v_t^-(f) = c_t(1 - a_3 - a_4\langle a^2\rangle f^2) \tag{4.6}$$

$$\gamma_l^+ = \frac{16\pi^2\lambda_6^2 \langle a\rangle f^2}{525\rho^2 c_l^6}, \quad \gamma_t^+ = \frac{2\pi^2\lambda_6^2 \langle a\rangle f^2}{75\rho^2 c_t^6} \tag{4.7}$$

$$v_l^+ = c_l(1 + a_5), \quad v_t^+ = c_t(1 + a_6) \tag{4.8}$$

$$a_1 = \frac{2\lambda_6^2}{375\rho^2 c_l^2}\left(\frac{2}{c_l^2} + \frac{3}{c_t^2}\right), \quad a_2 = \frac{2\pi\lambda_6^2}{6125\rho^2 c_l^2}\left(\frac{106}{c_l^4} + \frac{6}{c_l^2 c_t^2} + \frac{147}{c_t^4}\right) \tag{4.9}$$

$$a_3 = \frac{\lambda_6^2}{250\rho^2 c_t^2}\left(\frac{2}{c_l^2} + \frac{3}{c_t^2}\right), \quad a_4 = \frac{3\pi\lambda_6^2}{6125\rho^2 c_t^2}\left(\frac{49}{c_l^4} + \frac{2}{c_l^2 c_t^2} + \frac{75}{c_t^4}\right) \tag{4.10}$$

$$a_5 = \frac{2\lambda_6^2(4c_l^2 + c_t^2)}{525\rho^2 c_l^4(c_l^2 - c_t^2)}, \quad a_6 = -\lambda_6^2\frac{13c_t^2 + 7c_l^2}{2100\rho^2 c_t^4(c_l^2 - c_t^2)} \tag{4.11}$$

$$\lambda_6 = c_{11} - c_{12} - 2c_{44}, \quad \langle\alpha\rangle = {}^1/_5(c_{11} + 4c_{12} - 2c_{44}), \quad \langle\beta\rangle = {}^1/_5(c_{11} - c_{12} + 3c_{44}) \tag{4.12}$$

It is apparent from relations (3.8)-(4.12) that the absorption coefficients and wave velocities are calculated for the case of orthorhombic symmetry in terms of seven parameters ($P^2, b_1, \ldots, b_6$), each of which represents a quadratic function of the double-index elastic constants c_{ik}, for tetragonal symmetry in terms of four parameters ($\lambda_3, \ldots, \lambda_6$), for hexagonal symmetry in terms of three parameters ($\lambda_3, \ldots, \lambda_5$), and for cubic symmetry in terms of one parameter (λ_6), i.e., the number of indicated parameters is half the number of independent elastic constants c_{ik} for the given symmetry.

5. It is apparent from the foregoing results that the attenuation factor depends strongly on the polycrystalline grain sizes. In the long-wave case this dependence is determined by the factor $\langle a^3\rangle$, and in the short-wave case by $\langle a\rangle$. In order to compare the theoretical results with experimental, it is required to go from the variables $\langle a^3\rangle$ and $\langle a\rangle$ to appropriate experimentally measured quantities, namely, the average number of grains per unit area of the sample section or its equivalent characteristic, the average grain diameter in the plane of the section (image diameter). This problem is unsolvable in the general case. It is obvious, however, that the correlation scale α in the plane of the sample section is related to the volume correlation scale a by means of a certain numerical factor k. We then have

$$\langle a\rangle = k_1 k\alpha, \quad \langle a^2\rangle = k_2 k^2\alpha^2, \quad \langle a^3\rangle = k_3 k^3\alpha^3 \tag{5.1}$$

The coefficients k_i are as follows for exponential and Gaussian distributions of the coordinate part of the correlation functions [7], respectively:

$$k_1 = 1, \quad k_2 = 4\pi, \quad k_3 = 8\pi \quad (\varphi = \exp - r/a) \tag{5.2}$$

$$k_1 = {}^1/_2\sqrt{\pi}, \quad k_2 = 2\pi, \quad k_3 = \sqrt{\pi^3} \quad (\varphi = \exp - r^2/a^2) \tag{5.3}$$

We see from this result that the coefficients k_1 and k_3, which govern the short- and long-wave asymptotic behavior of the wave attenuation, can vary appreciably in transition from one structure to another. Thus, the transition from an exponential to a Gaussian dependence for $\varphi(r)$ induces a 4.5-fold variation of k_3. The coefficient k is also structure-sensitive. For an elementary structure such as the spheroidal graphite precipitations in pig iron [8] the ratio of the average grain diameter to the image diameter is 1.45. The same result has been obtained by computer calculations of quasispherical polycrystalline grains [9]. On the other hand, for grains of arbitrary configuration, needle-shaped for example, it is reasonable to expect the ratio of the average grain and image diameters to be different [8].

The foregoing numerical estimates indicate that we should expect agreement of the theoretical and experimental curves up to a constant factor depending on the structure. The value of the latter factor should be several units. The long-wave asymptotic behavior of the scattering coefficient has been compared with experiment in [6]. The theoretical equations used for verification in the latter paper are obtained from the expressions derived here for the long-wave asymptotic behavior of the scattering coefficient by letting $k_3k^3 = 24$ and $\alpha = r_{50}$, where r_{50} is the radius for which 50% of the grain images have a diameter smaller than r_{50}.

We point out that the comparison of the asymptotic behavior of long and short waves with experiment makes it possible to draw certain inferences regarding the value of the ratio k_3/k_1 and thus affords an indirect means of validating the choice of the function $\varphi(r)$.

LITERATURE CITED

1. I. M. Lifshits and G. D. Parkhomovskii, "Theory of ultrasonic wave propagation in polycrystalline media," Zh. Éksp. Teor. Fiz., 20, No. 2 (1950).
2. A. B. Bhatia and B. A. Moore, "Scattering of high-frequency sound waves in polycrystalline materials," J. Acoust. Soc. Am., 31, No. 8 (1959).
3. L. G. Merkulov, "Scattering of ultrasound in metals," Zh. Tekh. Fiz., 26, No. 1 (1956).
4. É. V. Bursian, V. V. Rychgorskii, and Ya. G. Girshberg, "Dispersion in barium titanate in the millimeter range above the transition temperature," Fiz. Tverd. Tela, 13, No. 2 (1971).
5. A. G. Fokin and T. D. Shermergor, "Correlation functions for the elastic field of quasi-isotropic solids," Prikl. Matem. i Mekhan., 32, No. 4 (1968).
6. E. P. Papadakis, "Ultrasonic attenuation caused by scattering in polycrystalline media," in: Physical Acoustics (W. P. Mason, ed.), Vol. 4B, Academic Press (1968), pp. 269-328.
7. A. A. Usov, A. G. Fokin, and T. D. Shermergor, "Dispersion of elastic waves in composition materials," in: Collected Scientific Papers on Microelectronics (Phys.-Math. Series) [in Russian], No. 3, Mosk. Inst. Élektron. Tekh., Moscow (1969).
8. S. A. Saltykov, Stereometric Metallography [in Russian], Metallurgizdat, Moscow (1958).
9. E. P. Papadakis, "From micrograph to grain-size distribution with ultrasonic applications," J. Appl. Phys., 35, No. 5 (1964).